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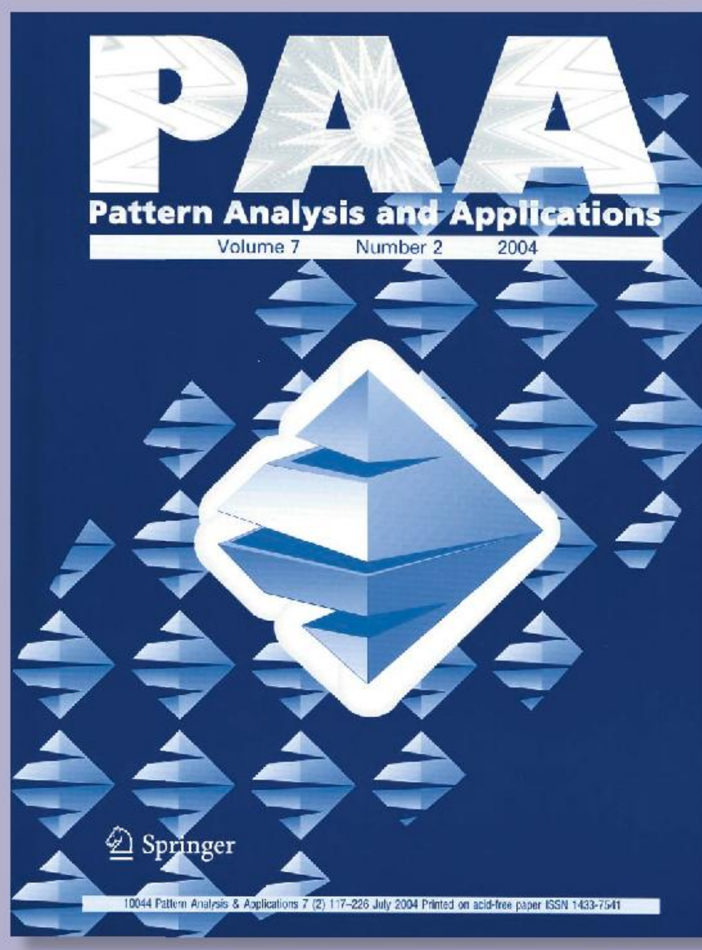

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THEORETICAL ADVANCES

Improving the non-extensive medical image segmentation based on Tsallis entropy

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Abstract Thresholding techniques for image segmentation is one of the most popular approaches in Computational Vision systems. Recently, M. Albuquerque has proposed a thresholding method (Albuquerque et al. in Pattern Recognit Lett 25:1059–1065, 2004) based on the Tsallis entropy, which is a generalization of the traditional Shannon entropy through the introduction of an entropic parameter q. However, the solution may be very dependent on the q value and the development of an automatic approach to compute a suitable value for q remains also an open problem. In this paper, we propose a generalization of the Tsallis theory in order to improve the non-extensive segmentation method. Specifically, we work out over a suitable property of Tsallis theory, named the pseudo-additive property, which states the formalism to compute the whole entropy from two probability distributions given an unique q value. Our idea is to use the original M. Albuquerque's algorithm to compute an initial threshold and then update the q value using the ratio of the areas observed in the image histogram for the background and foreground. The proposed technique is less sensitive to the q value and overcomes the M. Albuquerque and k-means algorithms, as we will demonstrate for both ultrasound breast cancer images and synthetic data.

Keywords Non-extensive entropy · Thresholding segmentation · Tsallis entropy

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1 Introduction

Segmentation is a fundamental step in image analysis. From a practical point of view, segmentation is the partition of an image into multiple regions (sets of pixels) according to some criterion of homogeneity of features such as color, shape, texture and spatial relationship. These fundamental regions are disjoint sets of pixels and their union compose the original whole scene [11]. Approaches in image segmentation can be roughly classified into (a) contour-based methods, like snakes and active shape models [7–10]; (b) region-based techniques [4, 20]; (c) global optimization approaches [6, 14]; (d) clustering methods, like k-means, fuzzy c-means, hierarchical clustering and EM [12]; and (e) thresholding methods [22].

Among these approaches, thresholding techniques are by far the most popular since they are simple but effective tools to separate objects from their backgrounds. The common approach to implement a thresholding technique is based on the image histogram by searching for local minima (valleys) of the gray-level histogram. Other possibility is to search for a threshold value constrained to the maximization of some information measure or entropy, like the classical Shannon one [15, 16]. This measure was introduced by Shannon in the information theory background, in his remarkable work [24]. Recently, Tsallis introduced another information measure to deal with specific phenomena in dynamical systems [27]. The Tsallis entropy is a generalization of the Shannon one and introduces a new parameter q, the so-called entropic parameter. Few years later, this entropy was applied by Albuquerque [2] for medical image segmentation, showing promising results. More recently, we proposed a generalization of this algorithm which was applied for breast cancer segmentation in ultrasound images [18]. Other interesting applications of thresholding

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segmentation are document analysis [1, 13], scene processing [3], and quality inspection of materials [21, 23]. A recent survey of image thresholding can be found in [22].

The new parameter q introduces a new degree of freedom which can be explored in order to improve robustness against factors such as noise and illumination changes [18, 19]. However, the solution may be very dependent on the q value and the development of an automatic approach to compute a suitable value for q remains an open problem.

In this paper, we propose a generalization of the Tsallis pseudo-additive property in order to improve the non-extensive segmentation method proposed by Albuquerque [2]. The proposed technique is less sensitive to the q value and overcomes the M. Albuquerque algorithm. Specifically, we work out over a suitable property of Tsallis theory, named the pseudo-additive property, which states the formalism to compute the whole entropy from two probability distributions given an unique q value. In our formalism, we associate a different q value for each probability distribution, providing greater robustness to the approach for thresholding estimation.

The idea proposed here is to use the original algorithm of Albuquerque [2] to compute an initial threshold value. Then, the q value is updated using the ratio of the areas under the histogram corresponding to the foreground and the background, respectively.

In order to validate our method we have carried out several experiments comparing the algorithms of Albuquerque [2] (as being the original method) and k-means (as a popular representative of clustering methods). These experiments are performed in ultrasound images and synthetic data as well.

The remainder of this paper is structured as follows: Sect. 2 introduces the idea of non-extensive entropy. In Sect. 3 we extend the non-extensive property and present our segmentation algorithm. Next, in Sects. 4 and 5 we present the experimental results. Finally, in Sect. 6, we present our final considerations.

2 Entropy and image segmentation

The entropy of a discrete source is often obtained from a probability distribution $P = [p_1, \ldots, p_k]$, $0 \le p_i \le 1$ under the restriction $\sum_i p_i = 1$, where p_i is the probability of finding the system in the state i. Under this formalism, the traditional and most popular form of entropy, called Shannon entropy, is computed as

$$S = -\sum_i p_i \ln(p_i) \tag{1}$$

Systems which statistics can be described by this entropy are called extensive systems because they have an additive property, defined as follows: let A and B be two random variables, with probability densities functions $P_A = [p_1, \ldots, p_n]$ and $P_B = [q_1, \ldots, q_n]$, respectively. If A and B are independent, then the entropy of the composed distribution[1] verify the so-called additivity rule:

$$S(A * B) = S(P_A) + S(P_B) \tag{2}$$

Recently, Tsallis introduced another information measure to deal with specific phenomena in dynamical systems, the so-called q-entropy [27], which is defined by

$$S_q(p_1, \ldots, p_k) = \frac{1 - \sum_{i=1}^{k} (p_i)^q}{q - 1} \tag{3}$$

where $p_i (i = 1, 2 \ldots, k)$ is the probability distribution and the real number q is the entropic parameter. A simple algebra shows that in the limit $q \to 1$, (3) meets the traditional Shannon entropy defined by (1). Tsallis entropy is not additive because, given two independent random variables A and B, we can show that [2, 24, 28]:

$$S_q(A * B) = S_q(A) + S_q(B) + (1 - q) \cdot S_q(A) \cdot S_q(B). \tag{4}$$

Therefore, the additive property does not hold unless in the limit $q = 1$. Systems whose statistics is governed by Tsallis entropy (or q-entropy) are called non-extensive, systems. As we said earlier, in the work presented by Albuquerque and colleges [2], the authors presented an approach to gray image segmentation based on the Tsallis formalism, carried out by (4). Suppose an image with k gray-levels, and let the probability distribution of these levels be $P = \{p_i = p_1; p_2; \ldots; p_k\}$. Now, we want to consider two probability distributions from P, one for the foreground (P_A) and another for the background (P_B). We can make a partition between the pixels from P into A and B. In order to maintain the constraints $0 \le P_A \le 1$ and $0 \le P_B \le 1$ we must to re-normalize both distributions as

$$P_A : \frac{p_1}{p_A}, \frac{p_2}{p_A}, \ldots, \frac{p_t}{p_A} \tag{5}$$

$$P_B : \frac{p_{t+1}}{p_B}, \frac{p_{t+2}}{p_B}, \ldots, \frac{p_k}{p_B} \tag{6}$$

where $p_A = \sum_{i=1}^{t} p_i$ and $p_B = \sum_{i=t+1}^{k} p_i$.

Now, we compute the a priori Tsallis entropy (Expression 3) for each distribution:

$$S_q(A) = \frac{1 - \sum_{i=1}^{t} \left(\frac{p_i}{p_A}\right)^q}{q - 1} \tag{7}$$

$$S_q(B) = \frac{1 - \sum_{i=t+1}^{k} \left(\frac{p_i}{p_B}\right)^q}{q - 1} \tag{8}$$

[1] We define the composed distribution, also called direct product of $P = [p_1, \ldots, p_n]$ and $Q = [q_1, \ldots, q_n]$, as $P * Q = \{p_i q_j\}_{i,j}$, with $1 \le i \le n$ and $1 \le j \le m$